Game Theory

Thinking Strategically I: Dominance & Common Knowledge

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		Them: A	Them: B
You	**A**	0, 0	2, −1
	B	−1, 2	1, 1

Example: The Prisoner's Dilemma

$$a > b > c > d$$

		2: l	2: r
1	**U**	b, b	d, a
	D	a, d	c, c

Example: The Prisoner's Dilemma

$a > b > c > d$

1 \ 2	l	r
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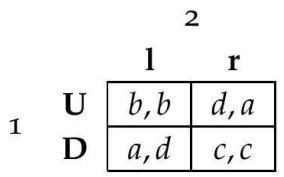
D	a, d	c, c

Example: The Prisoner's Dilemma

Dominance: For player 1, strategy **D** *strictly dominates* strategy **U**.
For player 2, strategy **r** *strictly dominates* strategy **l**.

$a > b > c > d$



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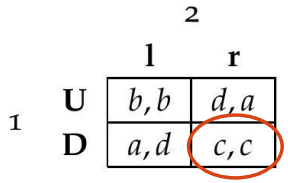
Outcome <**U**, **l**> *strictly* ***Pareto-dominates*** outcome <**D**, **r**>.



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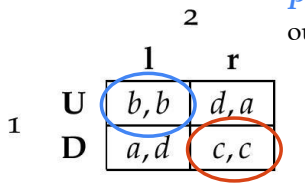


Example: The Prisoner's Dilemma

Dominance: For player 1, strategy **D** *strictly dominates* strategy **U**.
For player 2, strategy **r** *strictly dominates* strategy **l**.

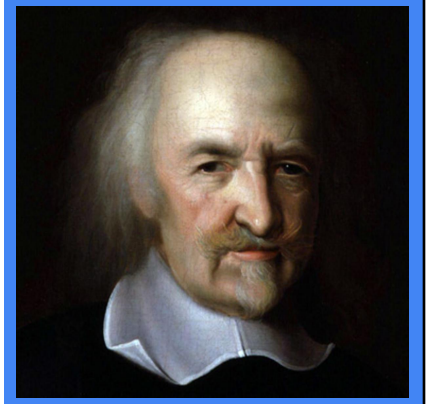
Life Lesson:

Sometimes, acting **rationally** can result in **a bad outcome**.

Example: The Prisoner's Dilemma

The Hobbesian State of Nature

What can the Prisoners' Dilemma teach us about Hobbes' view that the State of Nature = State of War?



Hobbes on the State of Nature

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	3rd / 3rd	1st / 4th
Relax	4th / 1st	2nd / 2nd

The State of Nature

Hobbes on the State of Nature

Mutual Peace
Mutual Aggression

Player 1 / Player 2	Prepare for violence	Relax
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Is this a Prisoners' Dilemma?

Hobbes on the State of Nature

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Is this a Prisoners' Dilemma?

Hobbes on the State of Nature

Mutual Peace
Mutual Aggression

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	3rd / 3rd	1st / 4th ∨?
Relax	4th / 1st	2nd / 2nd

Does everyone *really* prefer dominating over their neighbors?

Hobbes on the State of Nature

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Mutual Aggression

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What if you don't? What should you do then?

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Does everyone *really* prefer dominating over the neighbors?
What if you don't? **What should you do then**?

Hobbes on the State of Nature

Mutual Peace
Mutual Aggression

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	**3rd / 3rd** >	2nd / 4th
Relax	4th / 1st >	1st / 2nd

Lesson: Even if *you* most prefer Mutual Peace, if enough others prefer domination, it makes sense for you to prepare for violence too.

Hobbes on the State of Nature

Player 1 / Player 2	**Prepare for violence**	**Relax**
Prepare for violence	?? / ??	?? / ??
Relax	?? / ??	?? / ??

How might the game change under the Sovereign?

Hobbes on the State of Nature

Player 1 / Player 2	**Prepare for violence**	**Relax**
Prepare for violence	1 / 1	3 / 0
Relax	0 / 3	2 / 2

How might the game change under the Sovereign?

Hobbes on the State of Nature

Player 1 / Player 2	**Prepare for violence**	**Relax**
Prepare for violence	1 / 1	3 / 0
Relax	0 / 3	2 / 2

How might the game change under the Sovereign?
Introduce laws against **violence** (backed up by penalties).

Hobbes on the State of Nature

Player 1 / Player 2	**Prepare for violence**	**Relax**
Prepare for violence	? / ?	? / 0
Relax	0 / ?	2 / 2

How might the game change under the Sovereign?
Penalties change the payoffs. (Suppose the penalty = -2.)

Hobbes on the State of Nature

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	-1 / -1	1 / 0
Relax	0 / 1	2 / 2

How might the game change under the Sovereign?
Penalties change the payoffs. (Suppose the penalty = -2.)

Hobbes on the State of Nature

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	4th / 4th	2nd / 3rd
Relax	3rd / 2nd	1st / 1st

How might the game change under the Sovereign?
Penalties change the payoffs.

Hobbes on the State of Nature

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	4th / 4th	2nd / 3rd
Relax	3rd / 2nd	1st / 1st

How might the game change under the Sovereign?
Is this (still) a **Prisoners' Dilemma**?

Hobbes on the State of Nature

Player 1 / Player 2	Prepare for violence	Relax
Prepare for violence	4th / 4th	2nd / 3rd
Relax	3rd / 2nd	**1st / 1st**

How might the game change under the Sovereign?
Is this (still) a **Prisoners' Dilemma**? **No!**

Game Theory, in general

Game Theory

		2: l	2: r
1	U	b, b	d, a
	D	a, d	c, c

Game Theory

Players

		2: l	2: r
1	U	b, b	d, a
	D	a, d	c, c

Game Theory

Players

Strategies

		2: l	2: r
1	U	b, b	d, a
	D	a, d	c, c

Game Theory

Players

Strategies

Payoffs

		2: l	2: r
1	U	b, b	d, a
	D	a, d	c, c

Payoffs

Assign numbers to the outcomes, which represent how the players *rank* them.

Rankings:
- Complete
- Transitive

'Preference' / 'Utility'

$$a > b > c > d$$

Review: Strict Dominance

(Strict) Dominance

One strategy (e.g., "ask for an **A**") *strictly dominates* another (e.g., "ask for a **B**") just in case the payoff from the former is greater than the payoff of the latter **no matter what the other players choose.**

Test Your Understanding

Game Theory

Example:

		2		
		l	c	r
1	U	5,0	12,2	0,1
	D	7,5	0,2	1,1

Game Theory

Example:

		2		
		l	c	r
1	U	5,0	12,2	0,1
	D	7,5	0,2	1,1

Does Player 1 have a dominant strategy?

Game Theory

Example:

		2		
		l	c	r
1	U	5,0	12,2	0,1
	D	7,5	0,2	1,1

Does Player 1 have a dominant strategy?
Does Player 2 have a dominant strategy?

Game Theory

Example:

		2		
		l	c	r
1	U	5,0	12,2	0,1
	D	7,5	0,2	1,1

Does Player 1 have a dominant strategy?
Does Player 2 have a dominant strategy?

Example: Hannibal's Invasion

Game Theory

Example:

		2	
		n	s
1	N	1,0	0,2
	S	0,1	1,1

Player 1 chooses whether to defend **N**orth or **S**outh.
Player 2 chooses whether to attack from the **n**orth or the **s**outh.

Game Theory

Example:

		2	
		n	s
1	N	1,0	0,2
	S	0,1	1,1

If you were Player 1 what would you do?

Game Theory

Example:

		2: n	2: s
1	N	1,0	0,2
	S	0,1	1,1

Does Player 1 have a dominant strategy?

Game Theory

Example:

		2: n	2: s
1	N	1,0	0,2
	S	0,1	1,1

Does Player 1 have a dominant strategy?
Does Player 2 have a dominant strategy?

(Weak) Dominance

One strategy (e.g., "**s**") *weakly dominates* another (e.g., "**n**") just in case the payoff from the former is **always** as great as the payoff of the latter **no matter what the other players choose** and is sometimes greater.

Game Theory

Example:

		2: n	2: s
1	N	1,0	0,2
	S	0,1	1,1

For Player 2, strategy **s** *weakly dominates* strategy **n**.

Game Theory

Example:

		2	
		n	**s**
1	**N**	1, 0	0, 2
	S	0, 1	1, 1

For Player 2, strategy **s** *weakly dominates* strategy **n**.

Game Theory

Example:

		2	
		n	**s**
1	**N**	1, 0	0, 2
	S	0, 1	1, 1

For Player 2, strategy **s** *weakly dominates* strategy **n**.
So, what should Player 1 do?

The Number Game

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

Which number did you choose? Why?

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

Average: 24.25

The Number Game

Choose a number between 1 and 100.

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Average: 24.25

Magic number: 16.166...

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

Average: 24.25

Magic number: 16.166...

16 wins!

The Number Game

Choose a number between 1 and 100.

The number that is closest to two-thirds of the average wins the prize.

How might you approach playing this game?

The Number Game

How might you approach playing this game?

1. If picked randomly, average will be 50
2. 2/3rds of 50 = 33.3333

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What's wrong with this reasoning?

The Number Game

How might you approach playing this game?

1. If picked randomly, average will be 50
2. 2/3rd of 50 = 33.3333

What's wrong with this reasoning?

The numbers *weren't* picked at random!

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 100.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 100. Then, average = 100.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 100. Then, average = 100. So, Winning # = 66.66666

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 100. Then, average = 100. So, Winning # = 66.66666
So, picking 67 weakly dominates all higher numbers.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

But everyone else can work this out, too. So, no one will guess a number higher than 67.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 67. Then, average = 67. So, Winning # = 44.66666
So, picking 45 weakly dominates remaining strategies.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 45. Then, average = 45. So, Winning # = 30
So, picking 30 weakly dominates remaining strategies.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 30. Then, average = 30. So, Winning # = 20
So, picking 20 weakly dominates remaining strategies.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...Suppose everyone picked 20. Then, average = 20. So, Winning # = 13.333
So, picking 13 weakly dominates remaining strategies.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...and so on and so forth...

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...and so on and so forth…
Until we reach 1.

The Number Game

How might you approach playing this game?

Iteratively eliminate weakly dominated strategies!

...and so on and so forth…
Until we reach 1.

So, is the rational solution to guess 1?

The Number Game

So, is the rational solution to guess 1?

Not necessarily!

That argument made a strong assumption: *Common knowledge* of rationality.

Common Knowledge

Some fact (call it "*p*") is *common knowledge* just in case (1) everyone knows that *p*, (2) everyone knows that everyone knows that *p*, (3) everyone knows that everyone knows that everyone knows that *p*, ...

Questions?